The Clark Model for Business Cycle Analysis with Markov Switching Regimes

Master Thesis in Econometrics and Operations Research

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Outline of the presentation

- The business cycle. Pioneering works.
- The Clark model.
- Markov switching models.
- The Clark model with Markov switching regimes.
- Empirical results.
- Conclusions.

The analysis of the business cycle

- Burns & Mitchell (1946)
  - Business cycles are a type of **fluctuation** found in the aggregate economic activity.
  - A cycle consists of **expansions** occurring at about the same time in many economic activities, **followed by** [...] **recessions**, contractions, and revivals.
  - The **duration** of the business cycle varies from more than 1 year to 10-12 years.

The analysis of the business cycle

- The analysis of the business cycle involves the detection of **turning points** in an economic indicator.
- The **gross domestic product** (GDP) is broadly accepted as a measure of economic activity.

The analysis of the business cycle

- Beveridge & Nelson (1981)
  - Devised a procedure for the decomposition of a non-stationary time series into a **permanent** and a **transitory** component.
  - Two interpretations of the Beveridge-Nelson (BN) trend:
    - As an estimate of the trend in an **unobserved** component model.
    - As an **observed** component attached to the definition of an integrated process.

The Clark model

- Clark (1987)
  - Upon the framework of **structural time series models**,
  - a formal econometric model for the decomposition into a **permanent** plus a **transitory** components is specified as:

$$
\begin{aligned}
y_t &= n_t + x_t + u_t\,, & u_t &\sim NID(0, \sigma_u^2)\,,\\
n_t &= g_{t-1} + n_{t-1} + v_t\,, & v_t &\sim NID(0, \sigma_v^2)\,,\\
g_t &= g_{t-1} + w_t\,, & w_t &\sim NID(0, \sigma_w^2)\,,\\
x_t &= \phi_1 x_{t-1} + \phi_2 x_{t-2} + e_t\,, & e_t &\sim NID(0, \sigma_e^2)\,.
\end{aligned}
$$

Clark's model and further stylized facts

- In **Clark's model** the effect of shocks is **symmetric** throughout the phases of the cycle.

Further *stylized facts* observed:

- **Long and smooth expansion** periods alternate with **sharp and short recession** periods.
- The amplitude of a **recession** is **correlated** with the amplitude of the following **expansion**.

Markov switching models

- Hamilton (1989) proposed an autoregressive model with switching mean for the growth rates of the GDP:

$$y_t - \mu_{S_t} = \sum_{l=1}^{p} \phi_l (y_{t-l} - \mu_{S_{t-l}}) + \epsilon_t\,, \quad \epsilon_t \sim NID(0, \sigma^2)\,,$$
$$\mu_{S_t} = \mu_1 S_{1t} + \mu_2 S_{2t}\,,$$

where $S_{jt} = 1$ if $S_t = j$ and $S_{jt} = 0$ otherwise and S_t is a first order Markov process with transition probabilites:

$$Pr(S_t = j | S_{t-1} = i) = p_{ij}\,, \quad i, j = 1, 2\,.$$

Clark's model with Markov switching regimes

- **Asymmetries** in the business cycle can be modelled by means of the following variable:

$$\tau = \tau_0 + \tau_1 S_t \ ,$$

  with $S_t = 0$ in the first regime and $S_t = 1$ in the second regime. S_t is modelled as a first order Markov process with transition probabilities $Pr(S_t = j | S_{t-1} = i) = p_{ij}$ for $i, j = 1, 2$.

- Lam (1990) considers asymmetries in the trend component.

- Kim & Nelson (1999) consider asymmetries in the cycle component (Friedman's plucking model).

Clark's model with Markov switching regimes

- We implemment a general framework that encompasses previous models.
  - We take a state space representation for Clark's model.
  - Asymmetries in the components are modelled as a Markov switching variable.
  - Markov switching parameters are also considered.
  - The model is estimated by **approximate maximum likelihood** using Kim's filtering algorithm.
- The general setting is used to explore further versions of the Clark model with Markov switching regimes to address empirical questions in GDP series.

Empirical results: Switching damping factor

- A model with a switching damping factor as a measure of regime-dependent persistence of shocks.

$$
\begin{aligned}
y_t &= n_t + x_t + u_t \, , & u_t &\sim NID(0, \sigma_u^2) \, , \\
n_t &= n_{t-1} + \tau_0 + \tau_1 S_t \, , & & \\
x_t &= \rho_{S_t} \, x_{t-1} \cos\lambda + \rho_{S_t} \, x_{t-1}^* \sin\lambda + e_t \, , & e_t &\sim NID(0, \sigma_{e,S_t}^2) \, , \\
x_t^* &= -\rho_{S_t} \, x_{t-1} \sin\lambda + \rho_{S_t} \, x_{t-1}^* \cos\lambda + e_t \, , & e_t^* &\sim NID(0, \sigma_{e,S_t}^2) \, .
\end{aligned}
$$

The damping factor varies from one regime to the other:

$$
\rho_{S_t} = \rho_1 S_{jt} + \rho_2 S_{jt} \, ,
$$

where S_{jt} is an indicator variable that takes the value 1 when the j-th regime is governing the series and 0 otherwise. We consider two regimes, $j = 1, 2$.

Empirical results: Switching damping factor

FR GDP				US GDP			
p_{11}	0.866 (0.113)	τ_1	-0.174 (–)	p_{11}	0.993 (0.009)	τ_1	-0.095 (818.643)
p_{22}	0.952 (0.039)	τ_2	0.004 (0.035)	p_{22}	0.995 (0.005)	τ_2	0.172 (0.0290)
$\boldsymbol{\rho_1}$	**0.662** **(0.089)**	$\sigma^2_{e,1}$	0.039 (0.014)	$\boldsymbol{\rho_1}$	**0.910** **(0.027)**	$\sigma^2_{e,1}$	0.191 (0.044)
$\boldsymbol{\rho_2}$	**0.999** **(0.024)**	$\sigma^2_{e,2}$	0.120 (0.020)	$\boldsymbol{\rho_2}$	**0.959** **(0.020)**	$\sigma^2_{e,2}$	1.219 (0.148)
λ	0.148 (0.017)	σ^2_u	$0.8 \cdot 10^{-14}$ (–)	λ	0.131 (0.022)	σ^2_u	$0.8 \cdot 10^{-12}$ (0.026)
LL	-47.143				-271.265		

Empirical results: Correlated components

- Morley & Nelson & Zivot (2003)

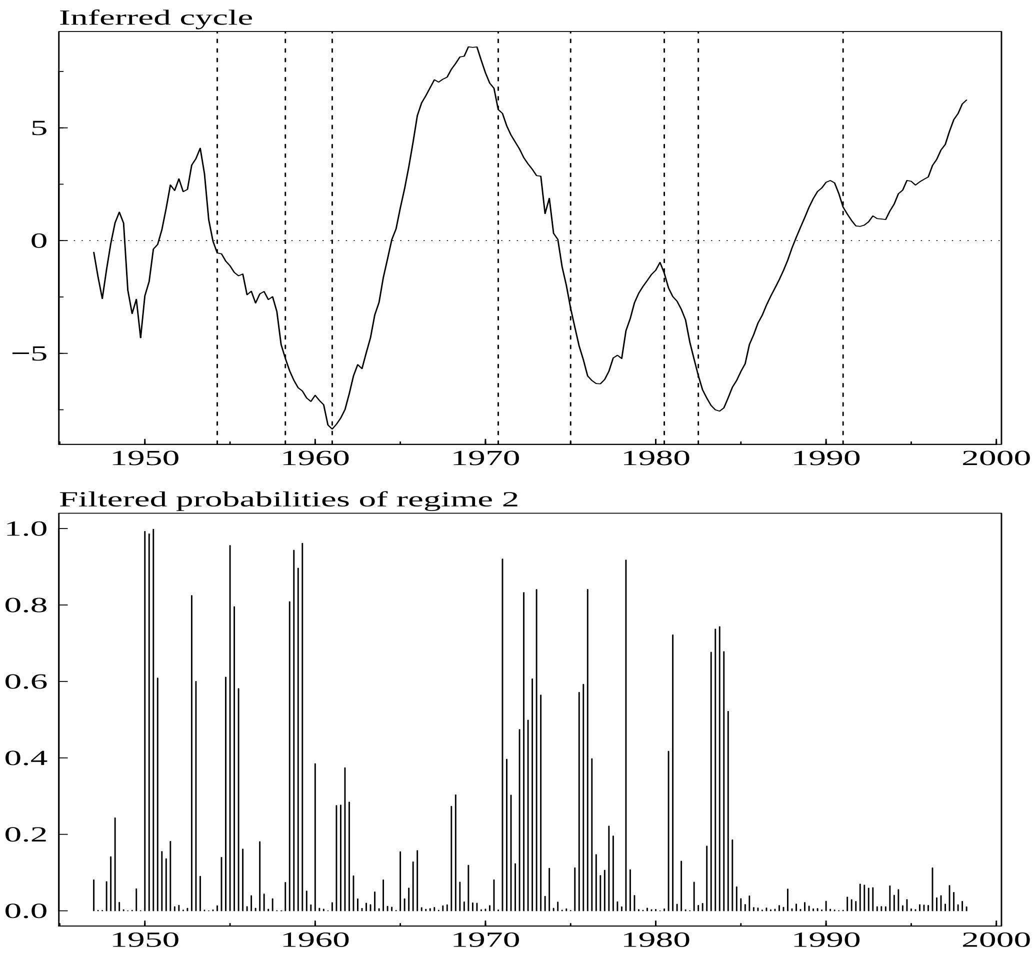
The following model with $\sigma_{ev} \neq 0$ is exactly identified:

$$
\begin{aligned}
y_t &= n_t + x_t \\
n_t &= n_{t-1} + \tau_0 + \tau_1 S_t + v_t\,, && v_t \sim NID(0, \sigma_v^2)\,, \\
x_t &= \rho x_{t-1} \cos\lambda + \rho x_{t-1}^* \sin\lambda + e_t\,, && e_t \sim NID(0, \sigma_e^2)\,, \\
x_t^* &= -\rho x_{t-1} \sin\lambda + \rho x_{t-1}^* \cos\lambda + e_t^*\,, && e_t \sim NID(0, \sigma_e^2)\,.
\end{aligned}
$$

Empirical results: Correlated components

US GDP			
p_{11}	0.935	p_{22}	0.634
	(0.030)		(0.120)
τ_1	-0.232	τ_2	1.789
	$(-)$		(0.308)
σ_v^2	$3.25 \cdot 10^{-9}$	σ_e^2	1.454
	$(-)$		(1.180)
$\sigma_{\mathbf{ev}}$	$\mathbf{-0.439}$	λ	0.100
	$(\mathbf{0.569})$		(0.022)
LL	-332.695		

Empirical results: Correlated components



Empirical results: Two transitory components

- A model with two transitory components:

$$y_t = n_t + x_t + u_t\ , \qquad u_t \sim NID(0, \sigma_u^2)\ ,$$

$$n_t = n_{t-1} + \tau_0 + \tau_1 S_t\ ,$$

$$x_t^{(1)} = \rho^{(1)} x_{t-1}^{(1)} \cos\lambda^{(1)} + \rho^{(1)} x_{t-1}^{(1)*} \sin\lambda^{(1)} + e_t^{(1)}\ , \qquad e_t \sim NID(0, \sigma_e^{2\,(1)})\ ,$$

$$x_t^{(1)*} = -\rho^{(1)} x_{t-1}^{(1)} \sin\lambda^{(1)} + \rho^{(1)} x_{t-1}^{(1)*} \cos\lambda^{(1)} + e_t^{(1)*}\ , \qquad e_t^{(1)} \sim NID(0, \sigma_e^{2\,(1)})\ ,$$

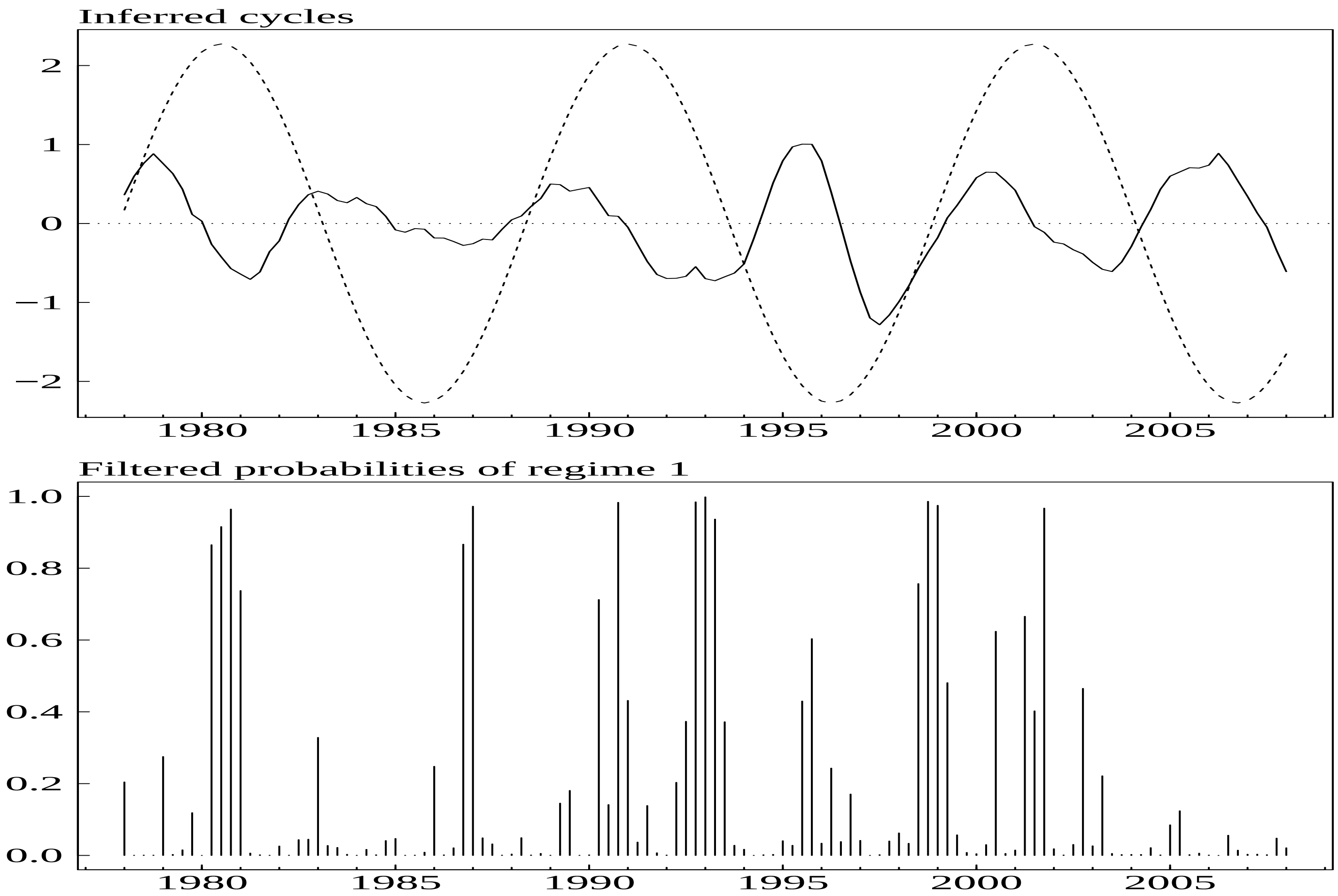
$$x_t^{(2)} = \rho^{(2)} x_{t-1}^{(1)} \cos\lambda^{(2)} + \rho^{(2)} x_{t-1}^{(2)*} \sin\lambda^{(2)} + e_t^{(2)}\ , \qquad e_t \sim NID(0, \sigma_e^{2\,(2)})\ ,$$

$$x_t^{(2)*} = -\rho^{(2)} x_{t-1}^{(1)} \sin\lambda^{(2)} + \rho^{(2)} x_{t-1}^{(2)*} \cos\lambda^{(2)} + e_t^{(2)*}\ , \qquad e_t^{(2)} \sim NID(0, \sigma_e^{2\,(2)})\ .$$

Empirical results: Two transitory components

FR GDP			
p_{11}	0.611	τ_1	-0.423
	(0.129)		(222.768)
p_{22}	0.913	τ_2	0.622
	(0.042)		(0.075)
$\lambda^{(\mathbf{1})}$	$\mathbf{0.298}$	$\sigma_e^{(1)}$	0.110
	$(\mathbf{0.017})$		(0.030)
$\lambda^{(\mathbf{2})}$	$\mathbf{0.150}$	$\sigma_e^{(2)}$	0.950×10^{-6}
	$(\mathbf{0.003})$		(0.0188)
Log-Lik.	45.406	σ_u	0.169
			(0.022)

Empirical results: Two transitory components



Conclusions

- In some cases, estimates in a model with changes of regime are in agreement with the understanding of the phases of the business cycle, while in other cases the non-linear model reveals the presence of a structural change or outlier observations.

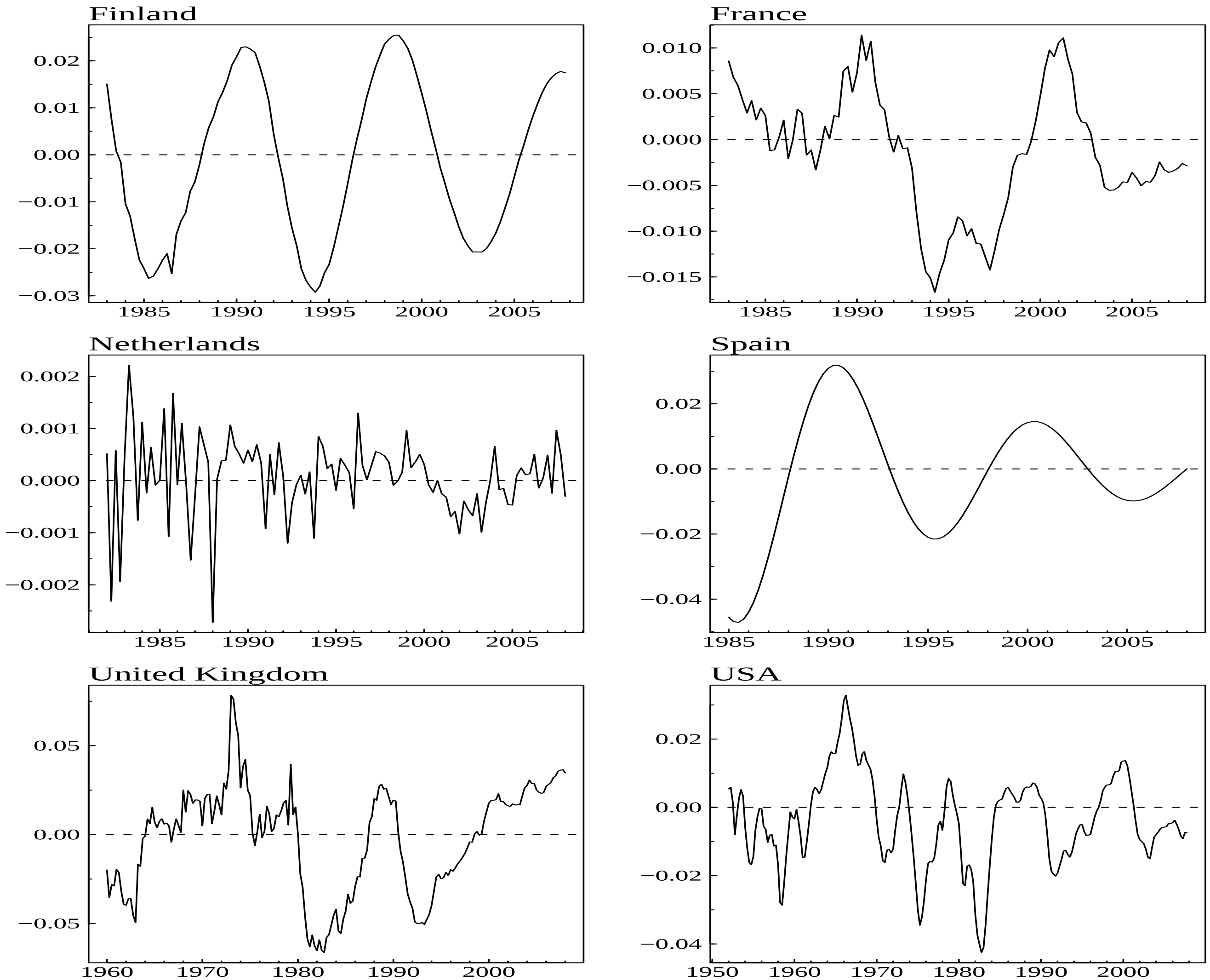
Conclusions

- The benchmark model discussed in this thesis provides a unified framework for the analysis of the business cycle.
- The Kim filtering algorithm is shown to be a useful tool for the estimation of a structural model with Markov switching regimes by approximate maximum likelihood.

Conclusions

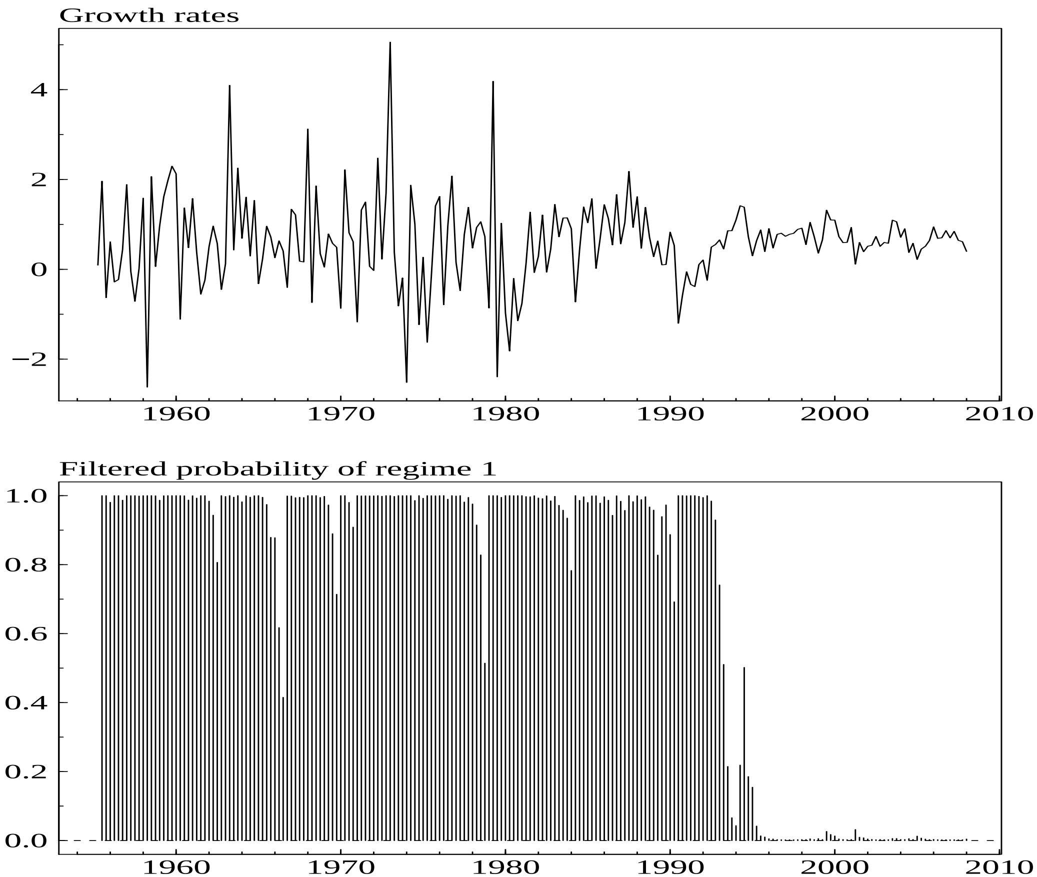
- A model with switching damping factor is estimated for the GDP of France and USA. Results for the GDP of France suggest the presence of lower persistence of shocks in the regime where a lower variance is estimated in the cyclical component.
- Correlation between the trend and the cyclical component in a model with a switch in the trend is estimated to be negative in the US GDP series.
- The presence of two transitory components is discussed in a model with asymmetries in the trend for the GDP of France. A deterministic cycle with periodicity 42 quarters and a stochastic cycle with periodicity 21 quarters are detected.

Empirical results: Clark's model



Empirical results: Markov switching model

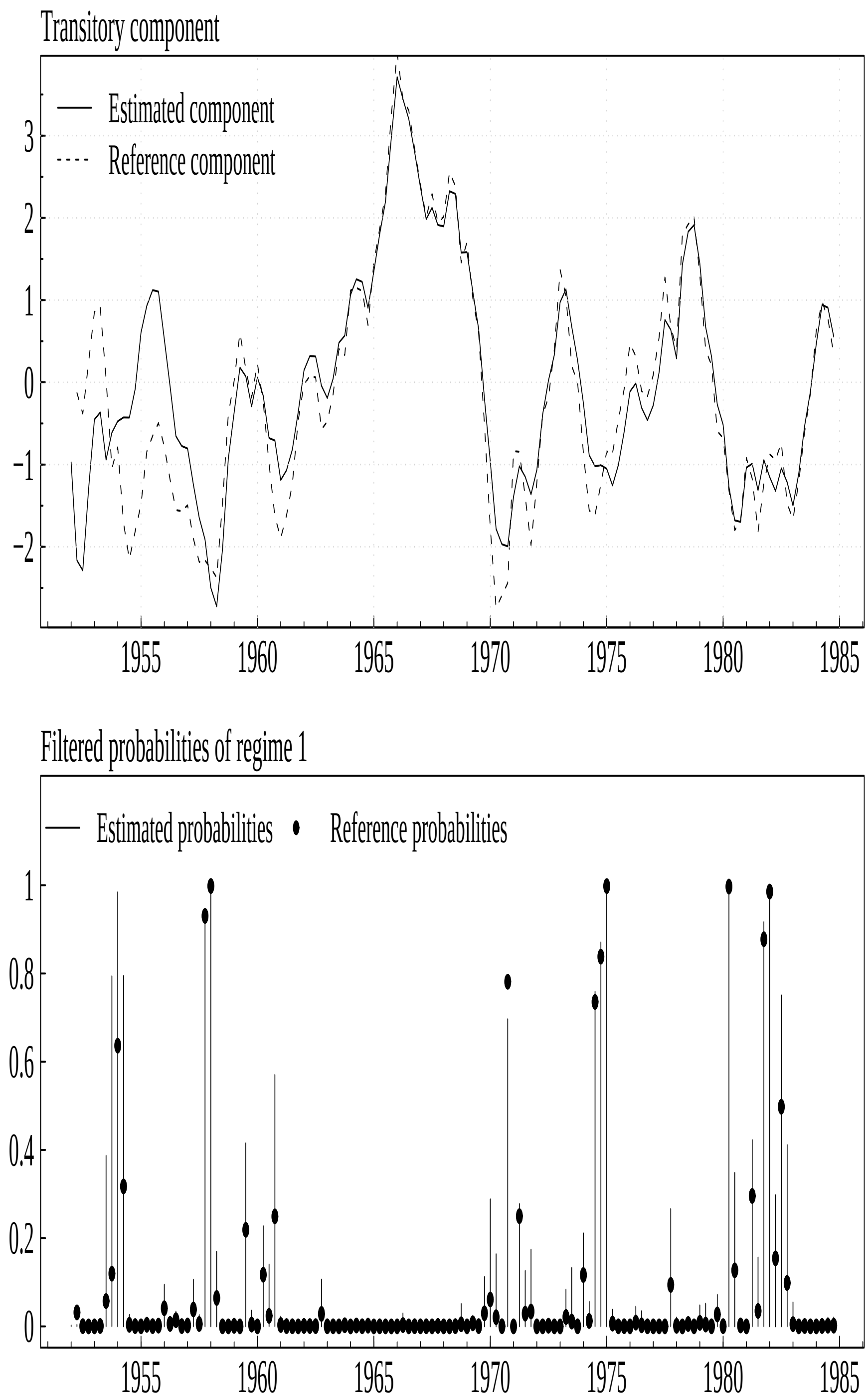
- UK GDP: AR model with Markov switching variance.



Approximate and exact ML

	Clark's model		Lam's model	
	Exact	Approx.	Exact	Approx.
p_{11}	-	-	0.508	0.560
p_{22}	-	-	0.957	0.932
σ_u	-	-	-	0.274
σ_v	0.0056	0.0056	0.771	0.620
σ_w	0.0002	0.0002	-	-
σ_e	0.0061	0.0061	-	-
τ_1	-	-	-1.483	-0.953
τ_2	-	-	2.447	1.924
ϕ_1	1.5346	1.5344	1.244	1.391
ϕ_2	-0.5888	-0.5884	-0.382	-0.484
Log-Lik.	578.52	578.54	-174.97	-180.33

Approximate and exact ML



State space representation

$$y_t = \begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} n_t \\ g_t \\ x_t \\ x_{t-1} \\ \vdots \\ x_{t-p+1} \end{bmatrix} + u_t \ ,$$

$$\begin{bmatrix} n_t \\ g_t \\ x_t \\ x_{t-1} \\ \vdots \\ x_{t-p+1} \end{bmatrix} = \begin{bmatrix} \tau_{S_t} \\ 0 \\ \delta_{S_t} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \phi_1 & \phi_2 & \cdots & \phi_p \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \ddots & \cdots & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} n_{t-1} \\ g_{t-1} \\ x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-p} \end{bmatrix} + \begin{bmatrix} v_t \\ w_t \\ e_t \\ 0 \\ \vdots \\ 0 \end{bmatrix} .$$

State space representation

$$y_t = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} n_t \\ g_t \\ x_t \\ x_t^* \end{bmatrix} + u_t \,,$$

$$\begin{bmatrix} n_t \\ g_t \\ x_t \\ x_t^* \end{bmatrix} = \begin{bmatrix} \tau_{S_t} \\ 0 \\ \delta_{S_t} \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \rho\cos\lambda & \rho\sin\lambda \\ 0 & 0 & -\rho\sin\lambda & \rho\cos\lambda \end{bmatrix} \begin{bmatrix} n_{t-1} \\ g_{t-1} \\ x_{t-1} \\ x_{t-1}^* \end{bmatrix} + \begin{bmatrix} v_t \\ w_t \\ e_t \\ e_t^* \end{bmatrix} .$$

$$\tau_{S_t} = \tau_0 + \tau_1 S_t \,,$$

$$\delta_{S_t} = \delta_0 + \delta_1 S_t \,,$$

S_t is an indicator variable modelled as a first order Markov process.

ML estimation

1. Run the Kalman filter for all the possible paths of the Markov process in the period t and $t-1$, $S_t = j, S_{t-1} = i$ with $i, j = 1, 2, ..., M$ and M the number of regimes. There are M^2 paths to consider leading to M^2 state values and variances.

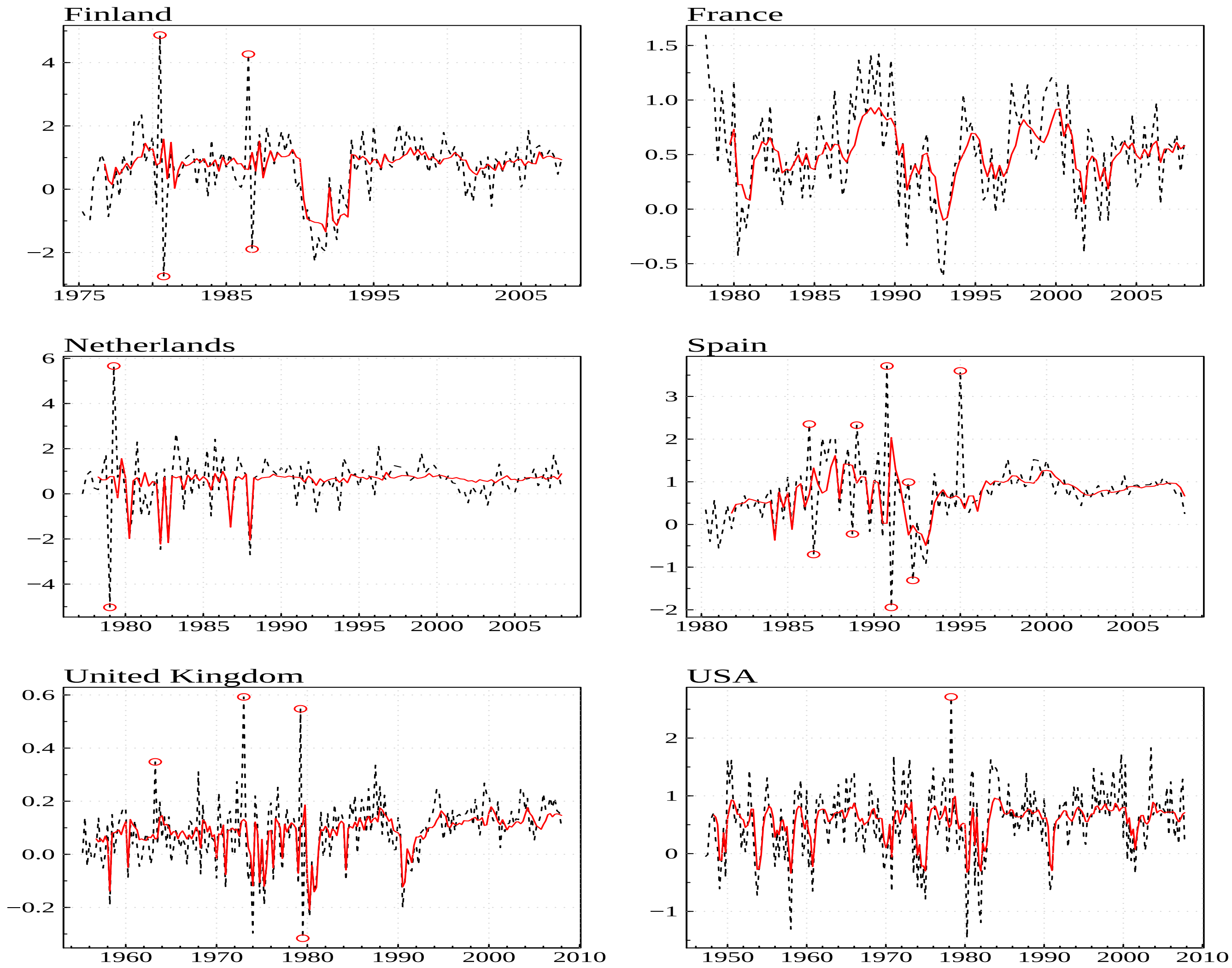
2. Run the Hamilton filter and compute the weighting terms $Pr(S_t, S_{t-1}|\psi_{t-1})$. The variable ψ_{t-1} denotes the set of information available up to time $t-1$.

3. Collapse the resulting M^2 state values and the corresponding variance covariance matrix (for each path $S_t = j, S_{t-1} = i$ with $i, j = 1, 2, ..., M$) into M-vectors according to the following approximations:

$$\alpha_{t|t}^{(j)} = \frac{\sum_{i=1}^{M} Pr\left(S_t = j, S_{t-1} = i|\psi_t\right) \alpha_{t|t}^{(i,j)}}{Pr\left(S_t = j|\psi_t\right)},$$

$$P_{t|t}^{(j)} = \frac{\sum_{i=1}^{M} Pr\left(S_t = j, S_{t-1} = i|\psi_t\right) \left(P_{t|t}^{(i,j)} + \left(\alpha_{t|t}^{(j)} - \alpha_{t|t}^{(i,j)}\right)\left(\alpha_{t|t}^{(j)} - \alpha_{t|t}^{(i,j)}\right)'\right)}{Pr\left(S_t = j|\psi_t\right)}.$$

Empirical results: Markov switching model



Empirical results: Markov switching model